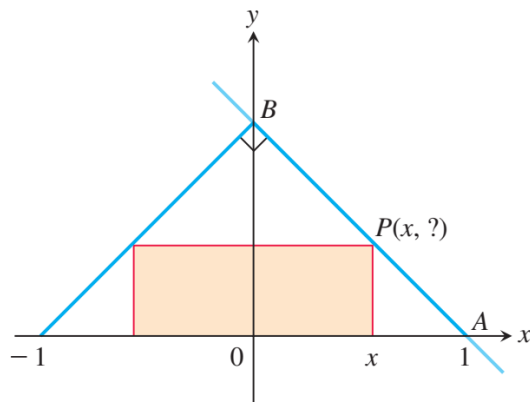


## Exercise 68

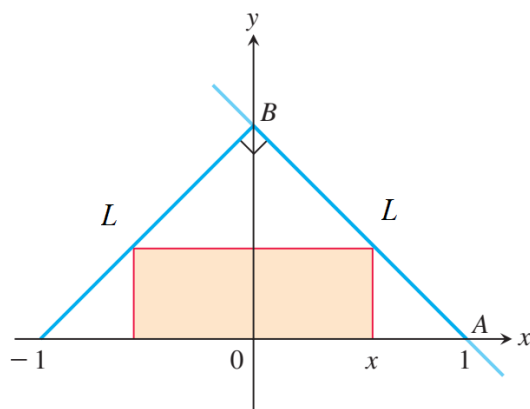
The accompanying figure shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.

- Express the  $y$ -coordinate of  $P$  in terms of  $x$ . (You might start by writing an equation for the line  $AB$ .)
- Express the area of the rectangle in terms of  $x$ .



### Solution

Because the triangle is isosceles, the two legs have the same length  $L$ .



And since the triangle is a right triangle, the Pythagorean theorem applies.

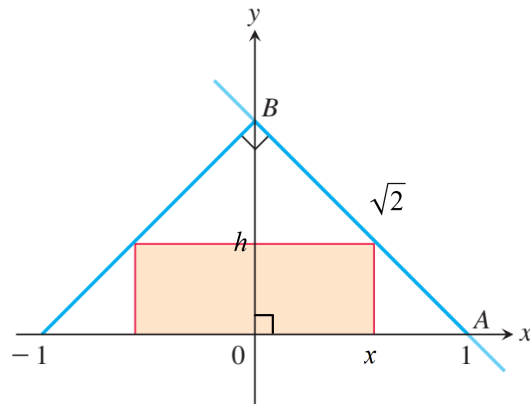
$$L^2 + L^2 = 2^2$$

$$2L^2 = 4$$

$$L^2 = 2$$

$$L = \sqrt{2}$$

Now the height of  $B$  can be determined using the Pythagorean theorem again.



$$1^2 + h^2 = (\sqrt{2})^2$$

$$1 + h^2 = 2$$

$$h^2 = 1$$

$$h = 1$$

Two points on line  $AB$  are  $(0, 1)$  and  $(1, 0)$ , so the equation of this line is

$$y = 1 - x.$$

This is the  $y$ -coordinate of  $P$ . The area of the rectangle is the product of its width and height.

$$\begin{aligned} A &= lw \\ &= (x + x)(y) \\ &= 2xy \\ &= 2x(1 - x) \\ &= 2x - 2x^2 \end{aligned}$$